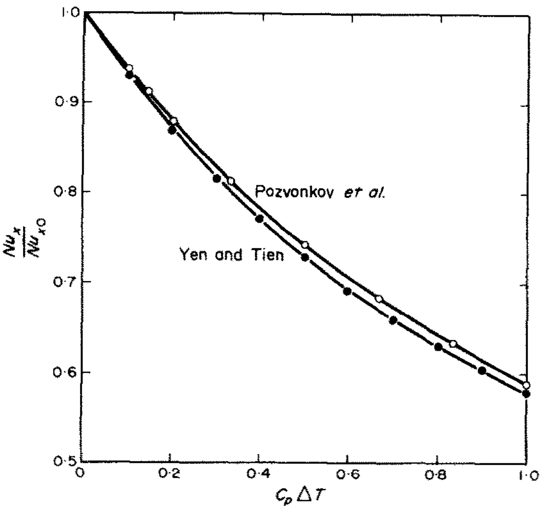

FIG. 1.

relates the local Nusselt number for heat transfer at a melting surface to that at a non-melting surface was derived.

$$\frac{Nu_x}{Nu_{x0}} = \sqrt{\left(\frac{a}{2}\frac{1}{1 + 1/k_f}\right)\left(\frac{\delta^{**}}{\delta_0^{**}}\right)} \qquad (1)$$

In our paper, the same problem was considered as a modified Leveque problem. Thus, the following expression was derived.

$$\frac{Nu_x}{Nu_{x0}} = \left(\frac{a_0}{a_N}\right)^{\frac{1}{3}} \qquad (2)$$

where

$$a_0 = \int_0^\infty \exp(-\lambda^3)\,d\lambda$$

$$a_N = \int_0^\infty \exp(-\lambda^3 + \beta\lambda/a_N)\,d\lambda$$

in which β is a dimensionless parameter defined as $C_p\Delta T/L$ (this is the reciprocal of the Kutateladze number as given in [1]) where C_p is the specific heat of water, ΔT is the temperature difference between the main water stream and the melting ice, and L is the latent heat of fusion. The value of a_N is unique and depends only on β. Figure 1 shows the comparison between equations (1) and (2).

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COMMENTS ON THE PAPER "CONVECTION NATURELLE TURBULENTE SUR UNE PLAQUE VERTICALE ISOTHERME, TRANSITION, ECHANGE DE CHALEUR ET FROTTEMENT PARIETAL, LOIS DE REPARTITION DE VITESSE ET DE TEMPERATURE"

(*Received* 10 *July* 1970)

IN A RECENT paper of Coutanceau [1] it is uncertain whether the turbulent boundary layer measured is fully developed or not. Figure 1 shows a comparison on the relation of Nu_x vs. Gr_x among the experimental results by Coutanceau and by Cheesewright [2] and a curve recommended by Fujii *et al.* [3]. The Grashof number corresponding to the

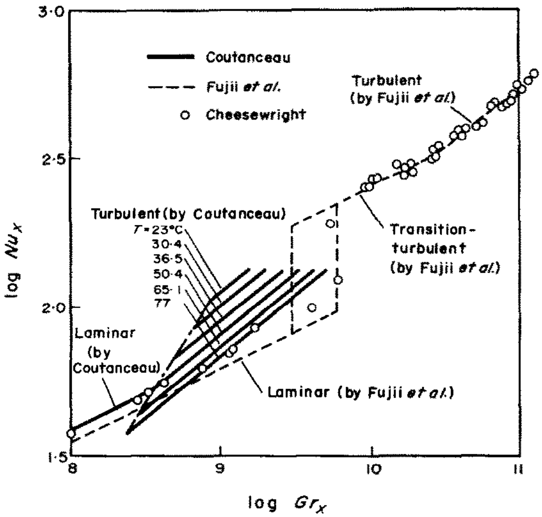

FIG. 1. Local coefficients of heat transfer.

upper limit of laminar region in Coutanceau's data is much lower than that in Cheesewright's. Moreover, even the uppermost Grashof number in Coutanceau's data is lower than the lowest one in the turbulent region of Cheesewright's. According to the experiments in air by Cheesewright and in liquids by Fujii *et al.* and Vliet-Liu [4], the local coefficients of heat transfer in the transitional region from laminar to turbulent increase abruptly. This characteristic, however, is not evident in Coutanceau's data, in which the turbulent region rather resembles the aforementioned transitional region.

Some examples of profiles of vertical velocity component and temperature are shown in Figs. 2 and 3 respectively, where the ordinates are

$$\bar{u} = \frac{u}{\sqrt{[g(T_p - T_0)x]/T_0}} = \frac{u^*}{\sqrt{[g(T_p - T_0)x]/T_0}}u^+,$$

$$\theta = \frac{T - T_0}{T_p - T_o} = 1 - \frac{T^+}{Z}$$

respectively, and the abscissa is

$$\zeta = Nu_x\frac{y}{x} = \frac{h}{\lambda}y = \frac{Pr}{Z}y^+.$$

Countanceau's curves are calculated from (4.1) to (4.11) and Table 1 in [1], Cheesewright's data are rearranged from Figs. 8 and 6 in [2], and the laminar profiles are referred from the numerical solutions of Ostrach [5]. Although the correlations of Cheesewright's data are very good, Coutanceau's data are scattered and closer to the laminar profiles rather than to the turbulent profiles of Cheesewright.

Coutanceau derives a new parameter gD_0^3/v_0^2 from the similitude analysis and confirms it experimentally. This result is unreasonable. The momentum equation corresponding to (1.1) in [1] must be rewritten as

$$\rho\frac{DV}{Dt} = -\,\boldsymbol{grad}\,p^* + (\rho_0 - \rho)g\,\boldsymbol{grad}\,x + \boldsymbol{F}_t,$$

where $p^* = p - (-\rho_0 g)$.

For natural convection along a vertical plate, the pressure term $\boldsymbol{grad}\,p^*$ is negligibly small and the buoyancy term

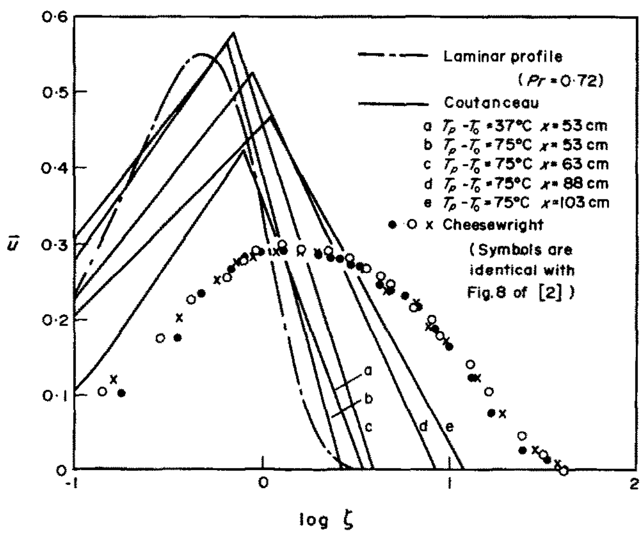

FIG. 2. Profiles of vertical velocity component.

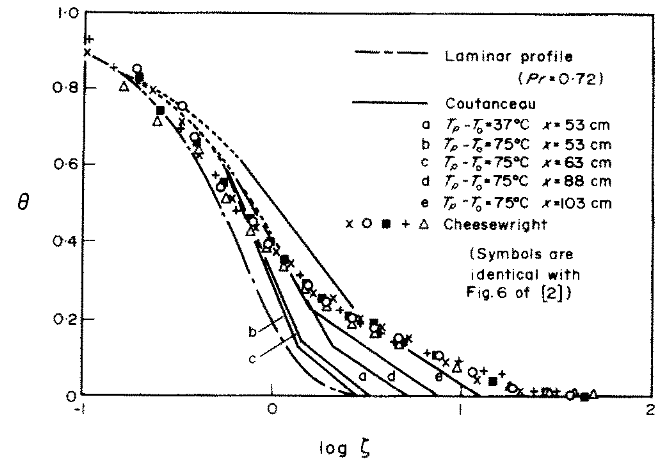

FIG. 3. Profiles of temperature.

$(\rho_0 - \rho) g\, \mathbf{grad}\, x$ becomes zero outside the boundary layer. The similitude analysis using above equation does not derive

$$\frac{gD_0^3}{\nu_0^2},$$

but

$$\frac{gD_0^3}{\nu_0^2} \frac{T_p - T_0}{T_0},$$

i.e. Grashof number.

Inclusively, Coutanceau mistakes the transitional region from laminar to turbulent for fully developed turbulent region. The turbulent flow in his experiment seems to be induced by the side effect of the heated plate, and the coefficient of heat transfer seems to be increased by both the turbulence induced and the end effect of the heated plate.

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